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Method for Determining Positions of Targets by Bistatic Measurements Using Signals Scattered by the Targets

The present invention relates to a method for determining positions of targets by bistatic measurements using signals scattered by the targets. Also the velocities of the targets can be determined. The method comprises a rapid bistatic association method which is suitable for, for instance, a network of radar stations in the manner of AASR (Associative Aperture Synthesis Radar) although there may be further fields of application. AASR is described, inter alia, in Swedish Patent 0101661-7, which is herewith incorporated by reference. In the following, the description will be concentrated on the new method of associating by bistatic measurements only.

First the fundamental problem that is solved by the invention will be presented. $N_s$ stations (for instance radar stations) are imagined to be set out in the space ($\mathbf{R}^3$). The stations are designated $s_j$, $j=1, \dots N_s$ and their position vectors are designated $\boldsymbol{\rho}_j$, $j=1, \dots N_s$. In addition to the stations, there are also $N_t$ moving targets which are to be detected. They are designated $t_i$, $i=1, \dots N_t$ and have corresponding time-dependent position vectors $\mathbf{r}_i = \mathbf{r}_i(t)$, $i=1, \dots N_t$.

Each station is capable of measuring distances (up to a certain maximum distance) and radial speed for each target. Thus, the station $s_j$, $1 \le j \le N_s$ will, at a certain point of time, measure

$$d_j(k) = |\mathbf{r}_k - \boldsymbol{\rho}_j|, \qquad k=1, 2, \dots N_{dj} \le N_t$$
$$v_j(k) = (d/dt)|\mathbf{r}_k - \boldsymbol{\rho}_j|, \quad k=1, 2, \dots N_{dj} \le N_t$$

For stations that are sufficiently close to each other, also bistatic measurement information is obtained, i.e. transmitting from one station and registration at another. For the pair of stations $(s_i, s_j)$ it means that the following is registered

$$d_{ij}(k) = |\mathbf{r}_k - \boldsymbol{\rho}_i| + |\mathbf{r}_k - \boldsymbol{\rho}_j| = d_i(k) + d_j(k), \quad k=1, 2, \dots \qquad N_{dij} \le N_t$$
$$v_{ij}(k) = (d/dt)|\mathbf{r}_k - \boldsymbol{\rho}_i| + (d/dt)|\mathbf{r}_k - \boldsymbol{\rho}_j| = v_i(k) + v_j(k), \quad k=1, 2, \quad N_{dij} \le N_t$$

It is to be noted that with these designations, $d_{ii}(k)=2d_i(k)$, $v_{ii}(k)=2v_i(k)$, $i=1,2, \dots$, $k=1,2, \dots$

For each sensor (monostatic or bistatic geometry) targets are thus registered in respect of distance and Doppler. It is a priori not possible to know which registration from one sensor is associated with a certain registration from another sensor, i.e. originating from the same target. If registrations from different sensors are paired incorrectly, false targets, ghost targets, arise. The problem of association is to discriminate, among all conceivable possibilities of combining sensor data, corresponding to conceivable target candidates, between correct combinations (targets) and false combinations (ghosts).

The maybe most straight-forward method is to consider three neighbouring stations and their monostatic registrations, which for the sake of simplicity are assumed to be N in number. These measurements can be combined in $N^3$ ways where each combination corresponds to a target position which is determined up to reflection in the plane containing the three stations. (Certain combinations can be incompatible, corresponding to false candidates.) These $\sim N^3$ candidates can then one by one be compared with the bistatic measurements and either be rejected or accepted. The problem of such a method is that it will be very slow if the number of targets, N, is large. For this reason, more efficient association algorithms have been developed.

Each target is to be determined in respect of position as well as velocity, i.e. they are to be positioned in a six-dimensional state space. The number of cells in the state space can be very large ($\sim 10^{18}$), which means that traditional projection methods will be irretrievably slow.

The above Swedish Patent 0101661-7 discloses a method of attacking the problem of association by designing a sensor network so that each target is registered by many sensors (monostatic and bistatic), i.e. a high degree of redundancy is obtained in the system. Then the state space is divided into a manageable number of relatively large cells.

If the cells are just large enough, it will be possible to reject many of them, i.e. they cannot contain any targets, for the following reasons. If the cell contains a target, all (or almost all) of the possible sensors that can register targets in the current cell, will indicate such a registration. On the other hand, if the cell is empty, some sensors, and yet not too many, will still indicate registrations (from other targets) that are compatible with the cell in question. Owing to redundancy, a sufficient number of

sensors will indicate the cell as empty, and it can be rejected. When the number of cells is thus reduced, the surviving cells are divided into smaller cells and the process is repeated. The process is repeated until the cells in the state space have reached the desired size. As the cells are becoming smaller, fewer and fewer ghost targets will survive, so that, when interrupting the process, practically only real targets are left. What speaks in favour of this method is that it uses (but also requires) the redundancy of the sensor network. However, it is not yet quite clear how rapid the method may eventually be.

An alternative method is disclosed in Swedish Patent 0101662-5, which is herewith incorporated by reference, and implies that use is made of certain symmetries of the combination sensors – measurement data. Given two stations, the two monostatic measurements, together with the bistatic measurement, will share a symmetry, viz. that the three measuring geometries are all insensitive to rotation of the targets about the axis extending through the two stations. This means that it is possible to make an initial rapid screening of the candidates and delete a large number of false associations (ghosts). The subsequent final association will then be significantly more rapid. A drawback, however, is that the monostatic measurements will be important, which may be disadvantageous in connection with reconnaissance of stealth targets.

The present method according to the invention is based on using a rapid method where only bistatic measurements are utilised. Furthermore the method manages a certain dropout of sensors in a better way than the method that has been discussed directly above. The method solves the current problem of association by being designed in the manner as is evident from the independent claim. Advantageous embodiments of the invention are defined in the remaining claims.

Before a more detailed description of the invention, first a multistatic network of ground radar stations will be contemplated, in which each radar station transmits radar pulses that are scattered towards flying targets and are then received by the surrounding stations. There will then be a situation involving a large number of bistatic measurements (i.e. the transmitting and the receiving station are located in different positions) and also monostatic measurements which, however, are not used in the invention. The bistatic measurements contain information about the total distance transmitter-target-receiver and corresponding Doppler information. A

coherent air situation image is then to be created from all these measurements. This problem, the problem of association, is non-trivial if there are a large number of targets.

For intuitive understanding of the invention, a simple case is taken into consideration, involving only one target, $m_1$, and four stations $s_1$, $s_2$, $s_3$, $s_4$. Now assume that the measurements $d_{12}$, $d_{34}$, $d_{13}$, $d_{24}$ are performed, where $d_{ij}$ means the total distance $s_i$ - $m_1$ - $s_j$. It will be appreciated that $d_{12} + d_{34} = d_{13} + d_{24}$ must be the case since both expressions mean the total distance from the target to the four stations.

If there are now N targets instead, the above observation can be used to correctly associate data in the following manner. All conceivable combinations of data of the type $d_{12}$ and $d_{34}$ are formed; they will be $N^2$ in number. In the same way, $N^2$ combinations of data of the type $d_{13}$ and $d_{24}$ are formed. These combinations are sorted and compared, and only sums from the two amounts that are equal (within a given tolerance) can correspond to real targets. The same discussion can be used about the Doppler velocities which thus give a further screening. In this way, quick and easy association of measurement data can be effected.

In general, the transmitters and receivers must be positioned and the range of the transmitters must be chosen so that a target at an arbitrary point within the position space can be measured via scattering in the target of at least four cooperating bistatic pairs of transmitters and receivers. The number of transmitters and receivers can be large. At least four such cooperating pairs are selected among these bistatic pairs to perform the association and the determination of the distance.

Below follows a more systematic presentation of the calculations. In order to obtain a simple description, the following (non-critical) assumptions are made. Assume that there are four stations and N targets, which all are seen by all sensors (monostatic as well as bistatic).

Input data is thus (monostatic measurements)

$$d_j(k) = |\mathbf{r}_k - \boldsymbol{\rho}_j|, \qquad k=1, 2, \dots N, \quad j=1,2,3,4$$

$$v_j(k) = (d/dt)|\mathbf{r}_k - \boldsymbol{\rho}_j|, \quad k=1, 2, \dots N, \quad j=1,2,3,4$$

and (bistatic measurements)

$$d_{ij}(k) = |\mathbf{r}_k - \boldsymbol{\rho}_i| + |\mathbf{r}_k - \boldsymbol{\rho}_j| = d_i(k) + d_j(k), \qquad k=1, 2, \ldots N, \quad j=1,2,3,4$$
$$v_{ij}(k) = (d/dt)|\mathbf{r}_k - \boldsymbol{\rho}_i| + (d/dt)|\mathbf{r}_k - \boldsymbol{\rho}_j| = v_i(k) + v_j(k), \qquad k=1, 2, \ldots N, \quad j=1,2,3,4$$

where i = j in the bistatic case corresponds to monostatic measurements, i.e. $i \neq j$ can be assumed if desirable.

It is to be noted that, for instance, for station j, with the monostatic measurement $d_j(k)$, k=1,2, ... N, it is not possible to know which measurement belongs to a certain target, i.e. the measurements are to be regarded as a set which is as a suggestion sorted according to distance. In this way, there is no connection between a certain index k which belongs to two different sensor registrations.

The method is now based on the following observation: For each registered target (not candidate, but real target) there must be a k, a k', an l and an l', all between 1 and N so that

$$d_{12}(k)+d_{34}(l)=d_{13}(k')+d_{24}(l')$$

For the same k, k', l, l', the following is also applicable

$$v_{12}(k)+v_{34}(l)=v_{13}(k')+v_{24}(l')$$

The reason is that if the target has the space vector $\mathbf{r}_t$, it is applicable for the target that

$$d_{12}(k)+d_{34}(l)= |\mathbf{r}_t - \boldsymbol{\rho}_1|+|\mathbf{r}_t - \boldsymbol{\rho}_2|+|\mathbf{r}_t - \boldsymbol{\rho}_3|+|\mathbf{r}_t - \boldsymbol{\rho}_4|$$

and in the same way that

$$d_{13}(k)+d_{24}(l)= |\mathbf{r}_t - \boldsymbol{\rho}_1|+|\mathbf{r}_t - \boldsymbol{\rho}_3|+|\mathbf{r}_t - \boldsymbol{\rho}_2|+|\mathbf{r}_t - \boldsymbol{\rho}_4|$$

so that they are equal. The argument for the velocities is identical. The suggested method now is as follows.

**Step 1**. Form the $N^2$ sums

$$d_{12}(k)+d_{34}(l), \quad 1 \leq l, \quad k \leq N$$

Sort them according to the total distance and designate them

$$d_{12+34}(m), \qquad 1 \leq m \leq N^2$$

**Step 2**. Proceed in the same way with

$$d_{13}(k')+d_{24}(l'), \quad 1 \leq l', \quad k' \leq N$$

so that the following will also be obtained (sorted)

$$d_{13+24}(m'), \qquad 1 \leq m' \leq N^2$$

**Step 3**. Associate targets from $\{ d_{12+34}(m)\}_{m=1,2\ldots N^2}$ with targets from $\{ d_{13+24}(m')\}_{m'=1,2\ldots N^2}$ if

$$| d_{12+34}(m)- d_{13+24}(m')| < \text{suitable tolerance}$$

**Step 4**. Investigate, and keep associated targets if they also satisfy

$$| v_{12+34}(m)- v_{13+24}(m')| < \text{suitable tolerance}$$

"Suitable tolerance" in Step 3 is determined, inter alia, by the transmitted signal bandwidth, the purpose of the processing and hypotheses about size and number of the targets. Usually it is from about one meter to some twenty or thirty meters. Correspondingly, "suitable tolerance" in Step 4 is usually a few meters/second.

To establish that this really results in a rapid method, the following rough estimate may be used. Assume that there are many targets so that they are of the same magnitude as the number of distance bins and the number of Doppler bins. This common number is again designated N. It may then be estimated that, since the total number of cells (= number of distance bins by the number of Doppler bins) is the same as the number of candidates in for instance $\{ d_{12+34}(m)\}_{m=1,2\ldots N^2}$, each such candidate

will be paired with typically a false candidate from $\{ d_{13+24}(m')\}_{m'=1,2...N^2}$. The number of candidates according to the above procedure thus is ~ $N^2$ (fewer with fewer targets), which is a great reduction compared with $N^3$. Further processing can then take place by comparing with the remaining bistatic geometry $\{d_{14+23}(m'')\}_{m''=1,2...N^2}$, the mono static measurements or measurements involving other stations.

It is also to be noted that it is possible to involve $\{d_{14+23}(m'')\}_{m''=1,2...N^2}$ from the beginning. This gives a possibility of having a redundancy, i.e. a possibility of managing a certain dropout in registrations, in the following way. The condition that $|d_{12+34}(m)- d_{13+24}(m')|$ < "suitable tolerance" can be seen as if both $d_{12+34}(m)$ and $d_{13+24}(m')$ are to be close to a certain given value. By requiring instead that two of $d_{12+34}(m)$, $d_{13+24}(m')$ and $d_{14+23}(m'')$ should be close to the indicated value (for some values of m, m' and m") there will still be a discrimination between false candidates (ghosts) and targets. However, it may be accepted that one of the measurements drops out.

The calculations in their entirety require $O(N^2 \log N)$ operations, and there are simple methods of really obtaining position and velocity from the candidates, i.e. after processing four bistatic distances are known for a certain candidate as follows:

$$
\begin{aligned}
|\mathbf{r} - \boldsymbol{\rho}_1| + |\mathbf{r} - \boldsymbol{\rho}_2| &= d_{12} \\
|\mathbf{r} - \boldsymbol{\rho}_3| + |\mathbf{r} - \boldsymbol{\rho}_4| &= d_{34} \\
|\mathbf{r} - \boldsymbol{\rho}_1| + |\mathbf{r} - \boldsymbol{\rho}_3| &= d_{13} \\
|\mathbf{r} - \boldsymbol{\rho}_2| + |\mathbf{r} - \boldsymbol{\rho}_4| &= d_{24}
\end{aligned}
$$

It is, of course, interesting to know the value of **r** (position of the target), i.e. a method of solving the above system of equations. ($\boldsymbol{\rho}_i$, i=1,2,3,4, are the known positions/ position vectors of the stations and $d_{12}$, $d_{34}$, $d_{13}$, $d_{24}$ are the measured bistatic distances.) Generally seen, intersections of ellipsoids cause relatively complicated algebraic systems of equations, but in this case the system of equations can be solved by simpler methods.

If the system of equations is regarded as a 4x4 system, it is obvious that it is degenerated. At the same time the condition $d_{12} + d_{34} = d_{13} + d_{24}$ guarantees that there is a parameter solution. By selecting the origin of coordinates in $\boldsymbol{\rho}_4$ so that

$|\mathbf{r} - \boldsymbol{\rho}_4| = |\mathbf{r}| = r$ and introducing r as a parameter, the following equations are obtained

$$|\mathbf{r}-\boldsymbol{\rho}_1| = d_{12} - d_{24} + r$$
$$|\mathbf{r}-\boldsymbol{\rho}_2| = d_{24} - r$$
$$|\mathbf{r}-\boldsymbol{\rho}_3| = d_{34} - r$$

It is here possible to square the three equations, in which case $r^2$ can be deleted, and obtain the following (for some a,b,c,α,β,γ)

$$\mathbf{r} \cdot \boldsymbol{\rho}_1 = ar + \alpha$$
$$\mathbf{r} \cdot \boldsymbol{\rho}_2 = br + \beta$$
$$\mathbf{r} \cdot \boldsymbol{\rho}_1 = cr + \gamma$$

The latter system of equations can then be solved in a fairly straight-forward way. However, there will be two different cases in dependence on whether $\{\boldsymbol{\rho}_i\}_{i=1,2,3}$ is linearly dependent or not.

The case of the velocities is similar, the following system of equations will be obtained

$$\begin{array}{ccccccccc} \hat{u}_1 \cdot \bar{v} & + & \hat{u}_2 \cdot \bar{v} & & & & & = & v_{12} \\ & & & & \hat{u}_3 \cdot \bar{v} & + & \hat{u}_4 \cdot \bar{v} & = & v_{34} \\ \hat{u}_1 \cdot \bar{v} & + & & & \hat{u}_3 \cdot \bar{v} & & & = & v_{13} \\ & & \hat{u}_2 \cdot \bar{v} & + & & & \hat{u}_4 \cdot \bar{v} & = & v_{24} \end{array}$$

where $\hat{u}_i = \dfrac{\bar{r} - \bar{r}_i}{|\bar{r} - \bar{r}_i|}$, $\bar{v} = \dot{\bar{r}}$, $i = 1,2,3,4$. The system of equations can be processed in the same fundamental way as the previous system of equations.

The invention can be implemented in high-level languages which are suitable for calculations, such as MatLab, C, Pascal, Fortran etc.